Computational Intelligence

Unit # 18

What is Fuzzy Logic

- Definition of Fuzzy
  - Fuzzy: “not clear, distinct, or precise; blurred”
- Definition of Fuzzy Logic
  - A form of knowledge representation suitable for notations that cannot be defined precisely but which depend upon their contexts.
- The term was coined by Lotfi Zadeh in 1965 with his mathematics of fuzzy set theory.

Successful Applications

- Automatic Control of dam gates for hydroelectric-power plants
- Camera aiming
- Compensation against vibration in camcorders
- Cruise-control for automobiles
- Controlling air-conditioning systems
- …..
- Many others

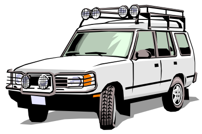
Examples of Linguistic Impression

- How was the weather like yesterday?
  - Oh! It was rainy with 98% humidity and hot with temperature of 35.5 deg C
  - Oh! It was very humid and really hot.

* Source: University Malaysian Pahang

Examples of Linguistic Impression (Cont'd)

- When you are at 10 meters from the junction start braking at 50% pedal level.



- When you are near the junction, start braking slowly.

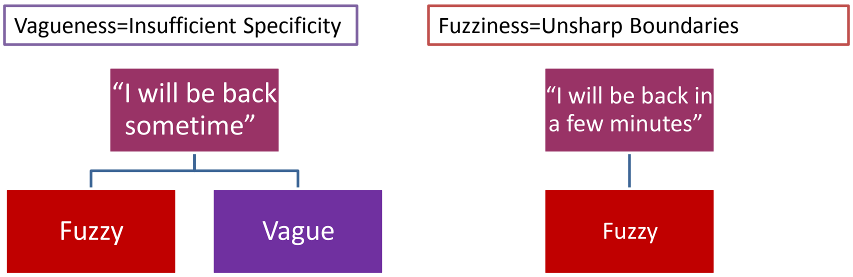
* Source: University Malaysian Pahang

Uncertainty vs Vagueness*

- Certainty – degree of belief
  - There is a 50% probability of rain today
  - I am 30% sure the patient is suffering from pneumonia
- Fuzziness – the degree to which an item belongs to a category
  - The man is tall
  - Move the wheel slightly to the left
  - The patient's lungs are highly congested

* Source: Susan Bridges @ Mississippi State University

Fuzziness Vs. Vagueness*

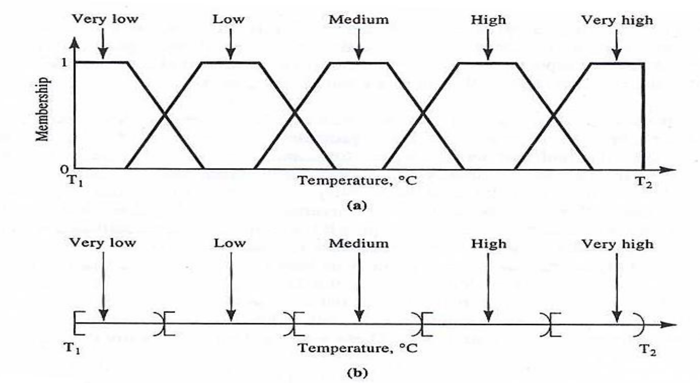


* Source: Raphael Steinberg @ Technion University

Bivalent Logic

- In classical logic, which is often described as Aristotelian logic, there are two possible truth values: propositions are either true or false.
- Such systems are known as **bivalent logics because they involve two logical values.**
- The logic employed in Bayesian reasoning and other probabilistic models is also bivalent: each fact is either true or false, but it is often unclear whether a given fact is true or false.
- Probability is used to express the likelihood that a particular proposition will turn out to be true.

Crisp vs. Fuzzy Variable

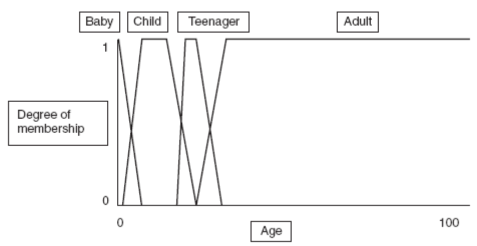


Temperature in the range $[T_1, T_2]$ conceived as: (a) a fuzzy variable; (b) a traditional (crisp) variable.

Example of a Fuzzy Variable

$$M_B(x) = \begin{cases} 1 - \frac{x}{2} & \text{for } x \le 2 \\ 0 & \text{for } x > 2 \end{cases}$$

$$M_C(x) = \begin{cases} \frac{x-1}{6} & \text{for } x \le 7 \\ 1 & \text{for } x > 7 \text{ and } x \le 8 \\ \frac{14-x}{6} & \text{for } x > 8 \end{cases}$$



- We represent a fuzzy set using a list of pairs, where each pair represents a value and the fuzzy membership value for that value.
- For example, we might define *B*, the fuzzy set of babies as follows:
  - *B* = {(0, 1), (2, 0)}
- Similarly,we could define the fuzzy set of children,*C*, as follows:
  - *C* = {(1, 0), (7, 1), (8, 1), (14, 0)}

Fuzzy Sets

- In traditional two-valued set theory, an element either belongs to a set or not. That is, set membership is precise.
- In fuzzy sets, an element belongs to a set to a degree, indicating the certainty (or uncertainty) of membership.

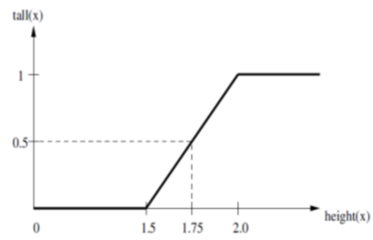
Membership Function

- The function is used to associate a degree of membership of each of the elements of the domain to the corresponding fuzzy set.
- Conditions
  - A membership function must be bounded from below 0 and from above by 1.
  - The range of a membership function must therefore be [0, 1].
  - For each $x \in X$, $\mu_A(x)$ must be unique. The is, the same element cannot map to different degrees of membership for the same fuzzy set.

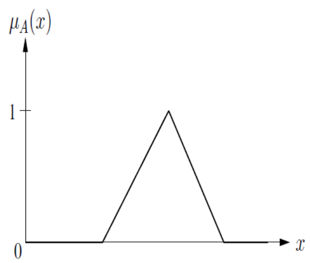
Illustration of *tall* Membership Function

- A possible membership function for *tall* fuzzy set can be defined as

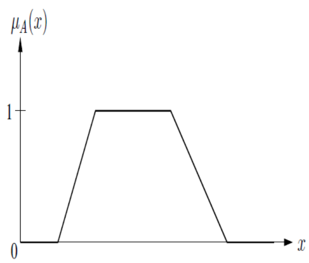
$$\text{tall}(x) = \begin{cases} 0 & \text{if length}(x) < 1.5 \\ (\text{length}(x) - 1.5) * 2 & \text{if } 1.5 < \text{length}(x) < 2 \\ 1 & \text{if length}(x) > 2 \end{cases}$$



Other Function Types



(a) Triangular Function



(b) Trapezoidal Function

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \le \alpha_{min} \\ \frac{x-\alpha_{min}}{\beta-\alpha_{min}} & \text{if } x \in (\alpha_{min}, \beta] \\ \frac{\alpha_{max}-x}{\alpha_{max}-\beta} & \text{if } x \in (\beta, \alpha_{max}) \\ 0 & \text{if } x \ge \alpha_{max} \end{cases}$$

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \le \alpha_{min} \\ \frac{x-\alpha_{min}}{\beta_1-\alpha_{min}} & \text{if } x \in [\alpha_{min}, \beta_1) \\ \frac{\alpha_{max}-x}{\alpha_{max}-\beta_2} & \text{if } x \in (\beta_2, \alpha_{max}) \\ 0 & \text{if } x \ge \alpha_{max} \end{cases}$$

Fuzzy Operators

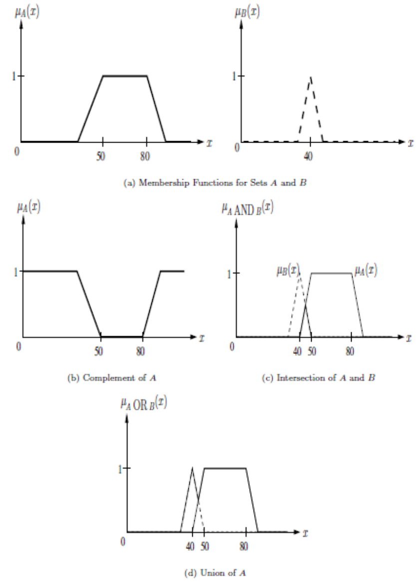
- **Equality:** Two fuzzy sets A and B are equal if and only if the sets have the same domain, and $\mu_A(x) = \mu_B(x)$ for all $x \in X$.

- **Complement of fuzzy set:** Let A^c denote the complement of set A. Then for all $x \in X$, $\mu_{A^c}(x) = 1 - \mu_A(x)$.

Fuzzy Operators (Cont'd)

- Intersection of fuzzy sets: If A and B are two fuzzy sets, then
  – Min-operator: $\mu_{A \cap B}(x)=\min\{\mu_A(x),\mu_B(x)\}, \forall x \in X$
  – Product Operator: $\mu_{A \cap B}(x)=\mu_a(x)\mu_B(x), \forall x \in X$
- Union of fuzzy sets: If A and B are two fuzzy sets, then
  – Max-operator: $\mu_{A \cup B}(x)=\max\{\mu_B(x),\mu_B(x)\}, \forall x \in X$
  – Summation Operator: $\mu_{A \cup B}(x)=\mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \forall x \in X$

Illustration of Fuzzy Operators



Exercise 1

Consider the two fuzzy sets:

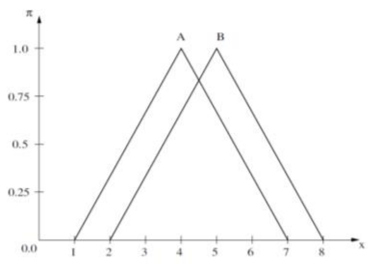
$$\text{long pencils} = \{pencil1/0.1, pencil2/0.2, pencil3/0.4, pencil4/0.6, pencil5/0.8, pencil6/1.0\}$$

$$\text{medium pencils} = \{pencil1/1.0, pencil2/0.6, pencil3/0.4, pencil4/0.3, pencil5/0.1\}$$

(a) Determine the union of the two sets.
(b) Determine the intersection of the two sets.

Exercise 2

- Consider the membership function of two fuzzy sets, A and B, as given in the figure.
  - Draw the membership function for the fuzzy set C = $A \cap B^c$, using the min-operator.
  - Compute $\mu_C(5)$.

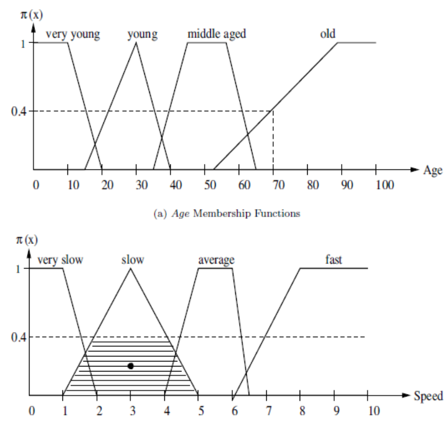


Fuzzy Reasoning

- A fuzzy reasoning system consists of three other components, each performing a specific task in the reasoning process:
  - Fuzzification
  - Inferencing
  - Defuzzification

Fuzzy Reasoning: Example 1

- Rule
  - If *Age* is *Old* the *Speed* is *Slow*
- What can be said about *Speed* if *Age* has the value of 70?



(a) *Age* Membership Functions

(b) *Speed* Membership Functions

Fuzzy Reasoning: Example 2

Let us suppose that we are designing a simple braking system for a car, which is designed to cope when the roads are icy and the wheels lock.

The rules for our system might be as follows:

Rule 1 IF pressure on brake pedal is medium
THEN apply the brake

Rule 2 IF pressure on brake pedal is high
AND car speed is fast
AND wheel speed is fast
THEN apply the brake

Rule 3 IF pressure on brake pedal is high
AND car speed is fast
AND wheel speed is slow
THEN release the brake

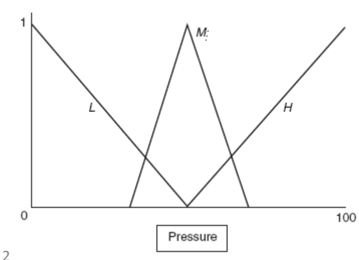
Rule 4 IF pressure on brake pedal is low
THEN release the brake

For this simple example, we will assume that brake pressure is measured from 0 (no pressure) to 100 (brake fully applied). We will define brake pressure as having three linguistic values: high (H), medium (M), and low (L), which we will define as follows:

$H = \{(50, 0), (100, 1)\}$

$M = \{(30, 0), (50, 1), (70, 0)\}$

$L = \{(0, 1), (50, 0)\}$



Example 2 (Cont'd)

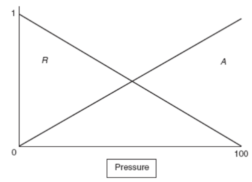
Similarly, we must consider the wheel speed. We will define the wheel speed as also having three linguistic values: slow, medium, and fast. We will define the membership functions for these values for a universe of discourse of values from 0 to 100:

$S = \{(0, 1), (60, 0)\}$

$M = \{(20, 0), (50, 1), (80, 0)\}$

$F = \{(40, 0), (100, 1)\}$

For the sake of simplicity, we will define the linguistic variable *car speed* using the same linguistic values (*S*, *M*, and *F* for slow, medium, and fast), using the same membership functions. Clearly, in a real system, the two would be entirely independent of each other.



Example 2: Fuzzification

- In a given situation, pressure value is 60, wheel speed is 55, and the car speed is 80.
  - $M_L(60) = 0.0$
  - $M_M(60) = 0.5$
  - $M_H(60) = 0.2$

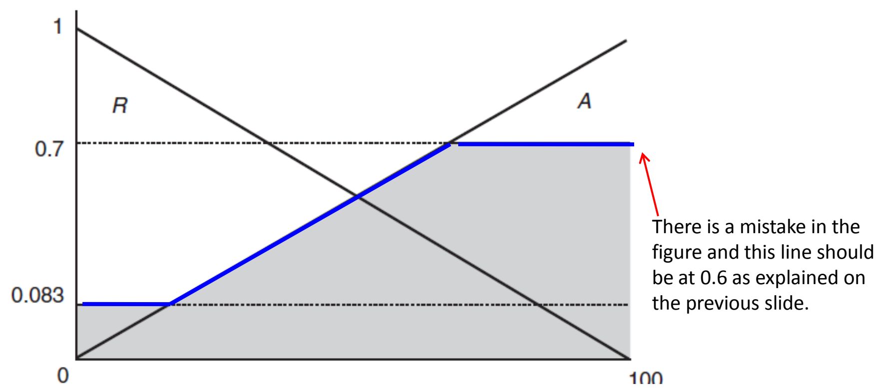
- $M_S(55) = 0.083$
- $M_M(55) = 0.833$
- $M_F(55) = 0.250$

- $M_S(80) = 0.0$
- $M_M(80) = 0.0$
- $M_F(80) = 0.667$

Example 2: Inferencing

- Fuzzy values obtained from the four rules are:
  - Rule 1: 0.5
  - Rule 2: Min(0.2, 0.667, 0.25) = 0.2
  - Rule 3: Min(0.2, 0.667, 0.083) = 0.083
  - Rule 4: 0
- Apply break should be 0.5 + 0.2 – 0.5*0.2 = 0.6
- Release break should be 0.083.

Example 2: Defuzzification



- Center of Gravity = (5 x 0.083 + 10x0.1 + 15x0.15 +……+ 70x0.6 + 75x0.6 + 80x0.6 + …. + 100x0.6)/ (0.083 + 0.1 + 0.15 + …… + 0.6)
- Center of Gravity = 63.97

Example 3

- Consider an imaginary medical system designed to recommend a dose of quinine to a patient or doctor based on the likelihood that that patient might catch malaria while on vacation.

Fuzzy Reasoning: Example 3

- Average temperature of destination (*T*)
- Average humidity of destination (*H*)
- Proximity to large bodies of water (*P*)
- Industrialization of destination (*I*)
- Dose of Quinine *(Q)*

$$M_{TH}(x) = \begin{cases} \frac{x-25}{75} & for\ x \geq 25 \\ 0 & for\ x < 25 \end{cases}$$

$$M_{HH}(x) = \frac{x}{100}$$

$$M_{TL}(x) = \begin{cases} 1-\frac{x}{75} & for\ x \leq 75 \\ 0 & for\ x > 75 \end{cases}$$

$$M_{HL}(x) = 1-\frac{x}{100}$$

Example 3: Membership Functions

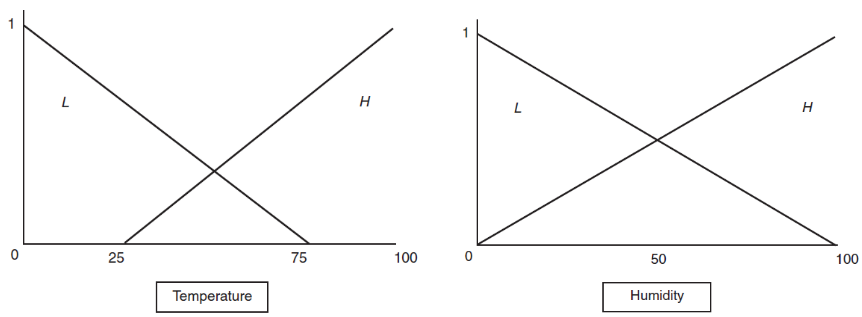
$$M_{PN}(x) = \begin{cases} 1 & for\ x < 10 \\ \frac{40 - x}{30} & for\ 10 \le x < 40 \\ 0 & for\ x \ge 40 \end{cases}$$

$$M_{IH}(x) = \begin{cases} 0 & for\ x < 10 \\ \frac{x - 10}{10} & for\ 10 \le x < 20 \\ 1 & for\ x \ge 20 \end{cases}$$

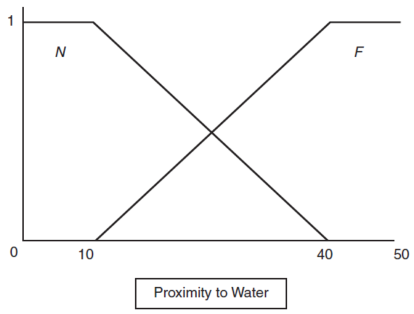
$$M_{PF}(x) = \begin{cases} 0 & for\ x < 10 \\ \frac{x - 10}{30} & for\ 10 \le x < 40 \\ 1 & for\ x \ge 40 \end{cases}$$

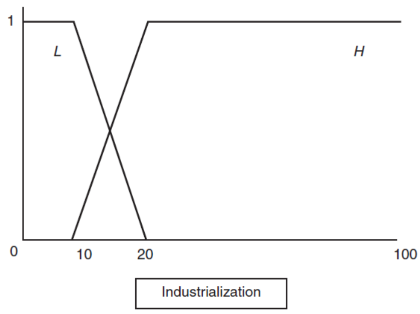
$$M_{IL}(x) = \begin{cases} 1 & for\ x < 10 \\ \frac{20 - x}{10} & for\ 10 \le x < 20 \\ 0 & for\ x \ge 20 \end{cases}$$

Example 3: Membership Functions (Cont'd)



Example 3: Membership Functions (Cont'd)



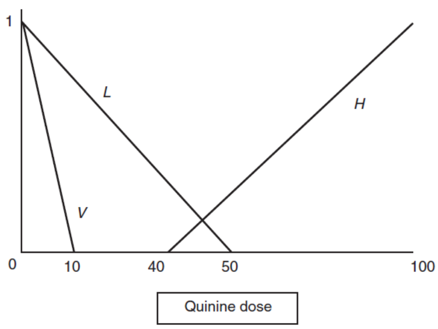


Example 3: Membership Functions (Cont'd)

$$M_{QV}(x) = \begin{cases} \dfrac{10-x}{10} & for\ x \le 10 \\ 0 & for\ x > 10 \end{cases}$$

$$M_{QL}(x) = \begin{cases} \dfrac{50-x}{50} & for\ x \le 50 \\ 0 & for\ x > 50 \end{cases}$$

$$M_{QH}(x) = \begin{cases} 0 & for\ x \le 40 \\ \dfrac{x-40}{60} & for\ x > 40 \end{cases}$$



V: Very Low Dose
L: Low Dose
H: High Dose

Example 3: Rules

- Rule 1
  - IF temperature is high
  - AND humidity is high
  - AND proximity to water is near
  - AND industrialization is low
  - THEN quinine dose is high
- Rule 2
  - IF industrialization is high
  - THEN quinine dose is low

Example 3: Rules (Cont'd)

- Rule 3
  - IF humidity is high
  - AND temperature is high
  - AND (industrialization is low
  - OR proximity to water is near)
  - THEN quinine dose is high
- Rule 4
  - IF temperature is low
  - AND humidity is low
  - THEN quinine dose is very low

Example 3: Input Data

- We will examine five sets of data, for five individuals, each of whom is traveling to a country that is at risk from malaria.
- The crisp data are as follows:
  - temperature = {80, 40, 30, 90, 85}
  - humidity = {10, 90, 40, 80, 75}
  - proximity to water = {15, 45, 20, 5, 45}
  - industrialization = {90, 10, 15, 20, 10}
- Hence, for example, person three is traveling to an area where the average temperature is 30, the humidity is 40, the distance to water is 20, and the level of industrialization is 15.

Example 3: Fuzzification

For traveler # 1

- M_{TH} (80) = 0.733
- M_{TL} (80) = 0

- M_{HH} (10) = 0.1
- M_{HL} (10) = 0.9

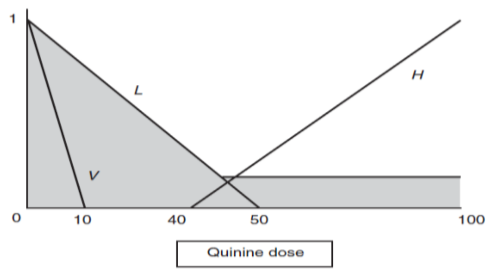
- M_{PN} (15) = 0.833
- M_{PF} (15) = 0.167

- M_{IH} (90) = 1
- M_{IL} (90) = 0

Example 3: Inferencing

- Rule 1: 0
- Rule 2: 1
- Rule 3: 0.1
- Rule 4: 0
- In other words
  - Very low dose (V) = 0
  - Low does (L) = 1
  - High dose (H) = 0.1

Example 3: Defuzzification



$$C = \frac{(0.9 \times 5) + (0.8 \times 10) + (0.7 \times 15) + (0.6 \times 20) + (0.5 \times 25) + (0.4 \times 30) + (0.3 \times 35) + (0.2 \times 40) + (0.1 \times 45) + (0.1 \times 50) + (0.1 \times 55) + (0.1 \times 60) + (0.1 \times 65) + (0.1 \times 70) + (0.1 \times 75) + (0.1 \times 80) + (0.1 \times 85) + (0.1 \times 90) + (0.1 \times 95) + (0.1 \times 100)}{0.9 + 0.8 + 0.7 + 0.6 + 0.5 + 0.4 + 0.3 + 0.2 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1}$$

$= 165 / 5.6$

$= 29.46$

Fuzzification for all 5 Cases

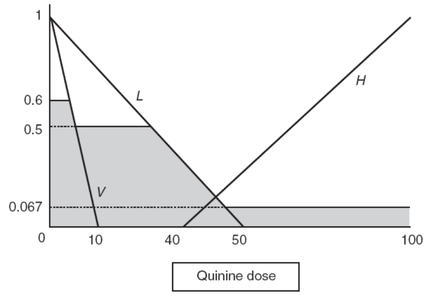
$$M_{TH} = \{0.733, 0.2, 0.067, 0.867, 0.8\}$$
$$M_{TL} = \{0, 0.467, 0.6, 0, 0\}$$
$$M_{HH} = \{0.1, 0.9, 0.4, 0.8, 0.75\}$$
$$M_{HL} = \{0.9, 0.1, 0.6, 0.2, 0.25\}$$
$$M_{PN} = \{0.833, 0, 0.667, 1, 0\}$$
$$M_{PF} = \{0.167, 1, 0.333, 0, 1\}$$
$$M_{IH} = \{1, 0, 0.5, 1, 0\}$$
$$M_{IL} = \{0, 1, 0.5, 0, 1\}$$

Inferencing for all 5 Cases

- Rule 1
  - (high dose): {0, 0, 0.067, 0, 0}
- Rule 2
  - (low dose): {1, 0, 0.5, 1, 0}
- Rule 3
  - (high dose): {0.1, 0.2, 0.067, 0.8, 0.75}
- Rule 4
  - (very low dose): {0, 0.1, 0.6, 0, 0}
- Since both Rule 1 and 3 suggest high dose, we can max of them. Thus
  - high dose: {0.1, 0.2, 0.067, 0.8, 0.75}

Example 3 (Cont'd)

- For Traveler 3
  - V: 0.6
  - L: 0.5
  - H: 0.067



$$C = \frac{(0.6 \times 5) + (0.5 \times 10) + (0.5 \times 15) + (0.5 \times 20) + (0.5 \times 25) + (0.4 \times 30) + (0.3 \times 35) + (0.2 \times 40) + (0.1 \times 45) + (0.067 \times 50) + (0.067 \times 55) + (0.067 \times 60) + (0.067 \times 65) + (0.067 \times 70) + (0.067 \times 75) + (0.067 \times 80) + (0.067 \times 85) + (0.067 \times 90) + (0.067 \times 95) + (0.067 \times 100)}{0.6 + 0.5 + 0.5 + 0.5 + 0.5 + 0.4 + 0.3 + 0.2 + 0.1 + 0.067 + 0.067 + 0.067 + 0.067 + 0.067 + 0.067 + 0.067 + 0.067 + 0.067 + 0.067 + 0.067}$$

$$= 128 / 4.3$$

$$= 29.58$$